NUCLEAR BREATHING MODE IN THE RELATIVISTIC MEAN-FIELD THEORY

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Abstract

The breathing-mode giant monopole resonance is studied within the framework of the relativistic mean-field (RMF) theory. Using a broad range of parameter sets, a systematic analysis of constrained incompressibility and excitation energy of isoscalar monopole states in finite nuclei is performed. A comparison is made with the incompressibility derived from the semi-infinite nuclear matter and with constraint nonrelativistic Skyrme Hartree-Fock calculations. Investigating the dependence of the breathing-mode energy on the nuclear matter incompressibility, it is shown that dynamical properties of surface respond differently in the RMF theory than in the Skyrme approach.

The breathing-mode giant monopole resonace (GMR) has been a matter of contention in the recent past [1]. The energy of the GMR has been considered to be a source of information on the nuclear matter compressibility. Empirically, data on the GMR have been measured with considerable precision [2]. The detailed analysis of the GMR leading to the incompressibility of nuclear matter is as yet not settled and is still under investigation. Theoretically, the incompressibility is understood to have been obtained [3] using the density-dependent Skyrme interactions. The deductions base themselves upon interpolation between various Skyrme forces for the GMR energies obtained from HF+RPA calculations. There has been, however, no unambiguous reproduction of the empirical GMR energies of medium-heavy nuclei using this approach. The breathing-mode energies depend not only upon the bulk incompressibility, but are also sensitive to the surface incompressibility. The relationship between the bulk and the surface incompressibility has been enunciated clearly for the Skyrme approach in Ref. [4].

The relativistic mean-field (RMF) theory has achieved a considerable success [5] in describing the ground-state properties of nuclei at and far away from the stability line. The dynamical aspects have, however, remained largely unexplored. The breathing-mode energies and incompressibilities were obtained within the RMF theory using the linear Walecka model [6,7]. The relationship of the GMR energies to the incompressibility of nuclear matter is not yet clear for the RMF theory. In the non-relativistic Skyrme approach, on the other hand, the GMR energies are connected to the incompressibility straightforwardly. In this letter, we examine the dependence of the breathing-mode energies in finite nuclei on the incompressibility of nuclear matter in the RMF theory.

We start from a relativistic Lagrangian [8] which treats nucleons as Dirac spinors ψ interacting by the exchange of several mesons: scalar σ-mesons that produce a strong attraction, isoscalar vector ω-mesons that cause a strong repulsion and isovector ρ-mesons required to describe the isospin asymmetry. Photons provide the necessary electromagnetic interaction. The model Lagrangian density is:

$$
\begin{aligned}
\mathcal{L} = {} & \bar{\psi}\{i\gamma_\mu\partial^\mu - M\}\psi + \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - U(\sigma) - g_\sigma\bar{\psi}\sigma\psi \\
& -\tfrac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi \\
& -\tfrac{1}{4}\vec{R}^{\mu\nu}\vec{R}_{\mu\nu} + \tfrac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - g_\rho\bar{\psi}\gamma^\mu\vec{\tau}\psi\vec{\rho}_\mu \\
& -\tfrac{1}{4}\vec{F}^{\mu\nu}\vec{F}_{\mu\nu} - e\bar{\psi}\gamma^\mu\frac{(1-\tau_3)}{2}\psi A_\mu,
\end{aligned}
\tag{1}
$$

where $U(\sigma)$ is the non-linear potential with the cubic and quartic terms [9]:

$$
U(\sigma) = \frac{1}{2}m_\sigma\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4. \tag{2}
$$

M, m_σ, m_ω and m_ρ are the nucleon, the σ-, the ω-, and the ρ-meson masses, respectively, and g_σ, g_ω, g_ρ and $e^2/4\pi$=1/137 are the coupling constants for the σ-, the ω-, the ρ-mesons

and for the photon. The field tensors for the vector mesons are $\Omega^{\mu\nu} = \partial^\mu \omega_\nu - \partial^\nu \omega_\mu$, $\vec{R}^{\mu\nu} = \partial^\mu \vec{\rho}_\nu - \partial^\nu \vec{\rho}_\mu - g_\rho(\vec{\rho}^\mu \times \vec{\rho}_\nu)$ and for the electromagnetic field $F^{\mu\nu} = \partial^\mu A_\nu - \partial^\nu A_\mu$. The variational principle leads to the stationary Dirac equation with the single-particle energies as eigenvalues,

$$\hat{h}_D \psi_i(x) = \varepsilon_i \psi_i(x), \tag{3}$$

where

$$\hat{h}_D = -i\vec{\alpha}. \triangledown + \beta(M + g_\sigma \sigma(r)) + g_\omega \omega^0(r) + g_\rho \tau_3 \rho^0(r) + e\frac{(1-\tau_3)}{2}A^0(r). \tag{4}$$

Solving these equations self-consistently one obtains the nuclear ground state in terms of the solution ψ_i.

In order to obtain the isoscalar monopole states in nuclei we perform constrained RMF calculations solving the Dirac equation

$$\left(\hat{h}_D - \lambda r^2\right)\psi_i(x, \lambda) = \varepsilon_i \psi_i(x, \lambda), \tag{5}$$

for different values of the Lagrange multiplier λ which keeps the nuclear rms radius fixed at its particular value

$$R = \left\{\frac{1}{A}\int r^2 \rho_\lambda(r) d^3r\right\}^{1/2}, \tag{6}$$

where

$$\rho_\lambda(r) \equiv \rho_v(r, \lambda) = \sum_{i=1}^{A} \psi_i^\dagger(x, \lambda)\psi_i(x, \lambda) \tag{7}$$

is the local baryon density determined by the solution $\psi_i(x, \lambda)$. The total energy of the constrained system

$$E_{RMF}(\lambda) = E_{RMF}[\psi_i(x, \lambda)], \tag{8}$$

is a function of λ (or the rms radius R) which has a minimum, the ground state energy $E_{RMF}^0 = E_{RMF}(0)$, at $\lambda = 0$ corresponding to the ground-state rms radius R_0.

This behaviour of the constrained energy (8) as a function of λ (or R) allows us to examine the isoscalar monopole motion of a nucleus as harmonic (breathing) vibrations changing the rms radius R around its ground-state value R_0. Considering $s = (R/R_0 - 1)$ as a dynamical collective variable and expanding (8) around the ground-state point s=0 (or $\lambda = 0$) we obtain in the harmonic approximation,

$$E_{RMF}(\lambda) = E_{RMF}^0 + \frac{1}{2}AK_C(A)s^2, \tag{9}$$

where

$$K_C(A) = A^{-1} \left(R^2 \frac{d^2 E_{RMF}(\lambda)}{dR^2} \right)_{\lambda=0}, \tag{10}$$

is the constrained incompressibility [10] of the finite nucleus. The second term in eq.(9) represents the restoring force of the monopole vibration. In order to obtain its associated inertial parameter we apply the method [6] of making a local Lorentz boost on the constrained spinors $\psi_i(x, \lambda)$:

$$\psi_i(x, t) = \frac{1}{\sqrt{\gamma}} \hat{S}(\vec{v}) \psi_i(x, \lambda), \tag{11}$$

where $\hat{S}(\vec{v})$ is the Hermitian local Lorentz boost operator

$$\hat{S}(\vec{v}) = \cosh\left(\frac{\phi}{2}\right) + \vec{\alpha}.\frac{\vec{v}}{|\vec{v}|} \sinh\left(\frac{\phi}{2}\right), \tag{12}$$

with $\phi = tanh^{-1}(|\vec{v}|)$ and $\gamma = \sqrt{1 - \vec{v}^2}$. The velocity field $\vec{v} = \vec{v}(\vec{r}, t)$ is then obtained by the continuity equation

$$\frac{\partial}{\partial t} \sum_{i=1}^A \psi_i^\dagger(x, \lambda)\psi_i(x, \lambda) + \bigtriangledown.\sum_{i=1}^A \psi_i^\dagger(x, \lambda)\vec{\alpha}\psi_i(x, \lambda) = 0, \tag{13}$$

which, using eqs.(7) and (11), transforms into the form

$$\dot{s}\, \frac{\partial \rho(r, \lambda)}{\partial s} + \bigtriangledown.\left(\rho(r, \lambda)\vec{v}(r, t)\right) = 0. \tag{14}$$

Up to first order in $\dot{s}$ the velocity field is determined by eq.(14) into the form $\vec{v} = -\dot{s}u(r)\vec{r}/|\vec{r}|$ where

$$u(r) = \left\{ \frac{\int_0^r \frac{d\rho(r', \lambda)}{ds} r'^2 dr'}{\rho(r, \lambda)r^2} \right\}. \tag{15}$$

The inertial parameter of the monopole vibration is then obtained as [6]

$$B_{rel}(A) = A^{-1} \int u^2(r)\, \mathcal{H}_{RMF}(r)\, d^3r, \tag{16}$$

where $u(r)$ is the velocity function (15) at $\lambda = 0$ and $\mathcal{H}_{RMF}(r)$ is the Hamiltonian density.

We have obtained $K_C(A)$ from eq.(10), $B_{rel}(A)$ from eq.(16) and the frequency of the isoscalar monopole vibration as

$$\omega_C = \sqrt{\frac{K_C(A)}{B_{rel}(A)}} \tag{17}$$

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for a number of spherical nuclei. Various parameter sets such as NL1 [11], NL-SH [5], NL2 [12], HS [13] and L1 [12] with values of the nuclear matter incompressibility $K_{NM} = 211.7$, 354.95, 399.2, 545 and 626.3 MeV, respectively, have been employed in the calculations. The last two sets, HS and L1, correspond to linear models without the self-coupling of the σ-field. In addition, the set L1 excludes the contribution from the ρ-field. Among the sets NL1, NL-SH and NL2 which correspond to the non-linear model, only the set NL2 has a positive coupling constant g_3, eq(2). Whereas the set NL1 reproduces the ground-state properties of nuclei only close to the stability line due to the very large asymmetry energy, the set NL-SH describes also nuclei very far away from the stability line [14].

Results from the present constrained RMF calculations are shown in Figs. 1 and 2, where the collective mass $B_{rel}(A)$, the constrained incompressibility $K_C(A)$ and the associated excitation energies ω_C for a few nuclei have been displayed. First, we consider $K_C(A)$ in Fig. 1 (a). The incompressibility of nuclei shows a strong dependence on the nuclear matter incompressibility K_{NM}, with a few exception for light nuclei. For the linear force HS, K_C shows a slight dip from the increasing trend for ^{208}Pb and ^{90}Zr, whereas for light nuclei ^{40}Ca and ^{16}O the HS values are even smaller than the NL2 values. The dependence of the imcompressibility $K(A)$ of finite nuclei on K_{NM} obtained from non-relativistic Skyrme calculations is different: it increases monotonically with K_{NM} [3]. This difference can be understood from the difference in the behaviour of the surface incompressibility in the two methods. In the Skyrme approach, the surface incompressibility has been shown to be $K_S \sim -K_{NM}$ for standard Skyrme forces [4]. However, this does not seem to be the case for the RMF theory as shown by the dip at the HS values. Thus, the surface incompressibility is not necessarily a straight function of the nuclear matter incompressibilty in the RMF theory.

Since the GMR energy depends strongly upon the inertial mass parameter, we next examine the mass parameter $B_{rel}(A)$ in Fig. 1 (b). The collective mass obtained in Eq. 16 from the velocity field shows an interesting dependence on the mass of the nucleus considered. For the heavy nucleus ^{208}Pb, the collective mass is about 0.9 and for ^{90}Zr it is 0.5 MeV^{-1}. For lighter nuclei it decreases to about 0.3 MeV^{-1}, as shown in Table 1. It does not, however, depend much upon the parameter set used and therefore shows only little sensitivity to K_{NM}. This implies that the K_{NM} dependence of the energy is then predominantly due to the K_{NM} dependence of the incompressibility $K_C(A)$.

We now compare in Table 1 the results from two RMF parameter sets, NL1 and NL-SH. The constrained incompressibilities K_C for NL-SH are higher than for NL1 as discussed above in Fig. 1(a). It is interesting to compare the relativistic collective mass $B_{rel}(A)$ with the expression usually applied in the non-relativistic sum-rule approach,

$$B_{sr}(A) = MR_0^2, \tag{18}$$

where the ground-state rms radius R_0 is used. The ratio $B_{sr}(A)/B_{rel}(A)$ is shown in Table

1 where the energy ω_C, eq.(17) and its approximation

$$\omega_{sr} = \sqrt{\frac{K_C(A)}{B_{sr}(A)}} \qquad (19)$$

have also been compared for the sets NL1 and NL-SH. The observation that the collective mass $B_{rel}(A)$ decreases significantly (up to 50 %) for light nuclei, provides a hint for the influence of the surface in the value $B_{rel}(A)$. For heavy nuclei the approximate value $B_{sr}(A)$ is rather close to $B_{rel}(A)$. Consequently, the monopole energies ω_C and ω_{sr} differ by about 1 MeV only for heavy nuclei and up to about 5 MeV for the lighter ones.

The energy ω_{sr} corresponds to the monopole excitation energy E_1 usually obtained from nonrelativistic Skyrme forces within the sum rule approach [15] and in nonrelativistic constrained Hartree-Fock calculations. In Table 1 energies ω_C and ω_{sr} have been compared with such nonrelativistic constrained HF results ω_{Sky} obtained from Eq. (19) with the Skyrme-type forces SkM and SIII having about the same nuclear matter incompressibility K_{NM} as the sets NL1 and NL-SH, respectively. From Table 1 it can be seen that the Skyrme results [4] ω_{Sky} are actually close to the energies ω_{sr} and differ significantly from the values of ω_C. This difference in the RMF constrained energy ω_C from the Skyrme constrained energy ω_{Sky} is small for heavy nuclei. It, however, increases for lighter nuclei, where the RMF shows smaller values. The difference is particularly significant for the higher compressibility forces (NL-SH and SIII). It arises naturally from the lower values of the collective mass $B_{rel}(A)$ for light nuclei in the RMF theory as discussed above. Thus, the masses B_{rel} and B_{sr} have different dependences on surface.

Fig. 2 shows the constrained breathing-mode energy ω_C for different nuclei. For heavier nuclei ^{208}Pb and ^{90}Zr, ω_C shows a behaviour similar to that shown by K_C in Fig. 1 (a), as the collective mass does not show much change with incompressibility (Fig. 1.b). The stagnation in energy at NL2 and HS is reminiscent of the effect of surface compression as in Fig. 1 (a), thus reflecting the role played by the surface in the RMF theory. In lighter nuclei this effect is more transparent. For ^{40}Ca and ^{16}O, ω_C is not related to K_{NM} in a simple way due to the combined effect of K_C and the collective mass B_{rel}, where there is even a decrease in ω_C with K_{NM}. Employing schematic parameter sets, it was also shown [17] earlier that within the relativistic quantum hadrodynamics the surface compression responds differently than in the Skyrme ansatz.

A comparison of the empirical values of the GMR energies with ω_C is worthwhile. The empirical values for ^{208}Pb and ^{90}Zr are 13.9±0.3 and 16.4±0.4 MeV, respectively. For lighter nuclei the values are very uncertain. Systematics of the values for ^{208}Pb show that the empirical value is encompassed by the constrained calculations curve from K_{NM} = 300 - 400 MeV. The ^{90}Zr value is, however, overestimated by the corresponding curve by 2-3 MeV.

We have also carried out calculations of the incompressibility for finite nuclei by scaling the density using semi-infinite nuclear matter. The incompressibility K_A for a finite nucleus

can be written as:

$$K_A = K_{NM} + K_S A^{-1/3} + K_\Sigma \left(\frac{N-Z}{A}\right)^2 + \frac{3}{5}\frac{e^2}{r_0}\left(1 - 27\frac{\rho_0^2 e'''}{K_{NM}}\right). \tag{20}$$

where major contribution to K_A arises from the volume (K_{NM}) and the surface terms (K_S). The surface incompressibility K_S has been obtained by calculating the second derivative of the surface tension (σ) with respect to the density for each change in the scaled density as given by [3]

$$K_S = 4\pi r_0^2 \{22\sigma(\rho_0) + 9\rho_0^2 \sigma''(\rho_0) + \frac{54\,\sigma(\rho_0)}{K_\infty}\rho_0^3 e'''_\infty(\rho_0)\}, \tag{21}$$

The details of the procedure to perform scaling of the density have been discussed in ref [16]. The last two terms in eq. (20) contribute very little. We add these terms for completeness, however. The asymmetry coefficient K_Σ has been taken at -300 MeV from the empirical determination [18] and is a reasonable value. The third derivative of the EOS, $\rho_0^2 e'''$, for each force is known from the nuclear matter calculations. We have performed the calculations with the parameter sets used above.

Fig. 3 shows the 'scaling' incompressibility K_A for various RMF forces obtained from Eq. (20). The general trend of the scaling incompressibility with K_{NM} is about the same as in Fig. 1(a), showing a dip for the force HS. In general, the K_A values are about 20% larger than the constrained values K_C. This is consistent with the known relationship of the two types of the incompressibilities for nuclear matter, whereby it was shown [10] that $K_A(NM) = \frac{7}{10}K_C(NM)$. Fig. 3 brings out again the importance of the role of the surface in the dynamical calculations within the RMF theory. The surface incompressibility K_S for the forces NL1 and NL-SH are -333.1 and -610.1 MeV respectively. Thus, in both the cases the ratio of the surface to the bulk incompressibility is obtained as 1.58 and 1.72 respectively. These values differ considerably from the ratio of 1 in the Skyrme ansatz. The values of the bulk and surface incompressibilities for NL-SH are closer to the empirical values from Ref. [2].

In conclusion, it has been shown that within the RMF theory, the incompressibility of a finite nucleus, $K(A)$, does not depend on the nuclear matter incompressibility in a simple way. This dependence is strongly related to the properties of the surface subjected to compression. This is different from the behaviour of the surface in the Skyrme ansatz. The collective mass for the breathing mode vibration in the RMF theory also shows a different behavior from light to heavy nuclei, thus affecting the frequency of the isoscalar breathing mode.

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TABLE I. The constrained incompressibility $K_C(A)$ in MeV, the mass parameter $B_{rel}(A)$ in $^{-1}$, the ratio $B_{sr}(A)/B_{rel}(A)$ and the associated monopole frequencies ω_C and ω_{sr}, (both in) calculated with the sets NL1 and NL-SH. Comparison is made with constrained Skyrme ts ω_{Sky} obtained within the sum-rule approach [15] with the Skyrme forces SkM and SIII.

| | NL1: $K_{NM} = 211.7\,\text{MeV}$ | | | | | | NL-SH: $K_{NM} = 354.95\,\text{MeV}$ | | | | | |
| | SkM: $K_{NM} = 216.7\,\text{MeV}$ | | | | | | SIII: $K_{NM} = 356.00\,\text{MeV}$ | | | | | |
ei	$K_C(A)$	$B(A)$	B_{sr}/B_{rel}	ω_C	ω_{sr}	ω_{Sky}	$K_C(A)$	$B(A)$	B_{sr}/B_{rel}	ω_C	ω_{sr}	ω_{Sky}
)	74.0	0.2282	0.742	18.0	20.9	22.4	105.6	0.2911	0.544	19.0	25.8	26.6
a	102.0	0.3086	0.894	18.2	19.2	20.2	153.4	0.3578	0.750	20.7	23.9	24.7
r	117.1	0.4776	0.924	16.2	16.3	17.0	149.0	0.4677	0.930	20.4	18.5	21.2
b	116.0	0.7825	0.991	12.2	12.2	12.9	197.7	0.8084	0.939	15.6	16.1	16.2

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FIGURES

FIG. 1. (a) The constrained incompressibility K_C in Eq. (10) for a few nuclei obtained using

bus RMF parameter sets. (b) The collective mass B_{rel} for the breathing-mode monopole

ations obtained from Eq. (16) within the RMF theory.

FIG. 2. The frequency ω_C of the monopole mode obtained using eq. (17).

FIG. 3. The incompressibility K_A (Eq. 20) obtained from 'scaling' of the nuclear density in

semi-infinite nuclear matter using the Thomas-Fermi approximation.